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Second law analysis of convective droplet burning

ISHWAR K. PURI
Department of Mechanical Engineering (M/C 251), University of Illinois at Chicago,
Box 4348, Chicago, IL 60680, U.S.A.

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Abstract—The entropy generation due to burning particles in a gaseous stream is considered and the contributions to it compared. A second law analysis is undertaken in order to minimize the entropy generation and, therefore, the lost available work. The optimum flow conditions from this thermodynamically advantageous perspective are determined for a burning droplet at low Reynolds number and an optimum transfer number obtained. The transfer number so obtained depends directly on the square of the relative velocity, and inversely on the net enthalpy rise due to burning and the ratio of ambient to flame temperature. In realistic flows, where the transfer number and net heat release are fixed, these quantities are related to the relative velocity and ambient to flame temperature ratio in order to operate at optimum conditions. The square of the relative velocity in such flows is a small fraction of the net heat release so that, to operate at optimum thermodynamic conditions, it is determined that the droplet Reynolds number must be large suggesting a large droplet size and low gas viscosity. Circumstances pertaining to engineering practice are also considered and it is concluded that within constraints practice is consistent with the implications of the second law analysis.

1. INTRODUCTION

THE USE of the second law of thermodynamics has found extensive application in problems involving heat transfer. Bejan [1–3] presents the methodology behind this approach as it is applied to heat and fluid flow, and reviews the literature in the area. San *et al.* [4], and Poulikakos and Johnson [5] present analyses for forced convection phenomena when irreversibilities due to mass transfer are important compared with those due to heat transfer and fluid friction. The key to analyses conducted by using the second law lies in evaluating the degree of irreversibility in engineering systems [6, 7] so as to determine the available work. The various constituents of the irreversibility (e.g. from heat and mass transfer, fluid friction, drag, and heat release) are then examined to understand their interrelationship and the mechanism of entropy generation in the flow. An optimum operating condition corresponding to a minimum entropy generation can be specified upon comparing these contributions.

Work, that would otherwise have been available, but is lost due to entropy production [2], termed as lost available work, is related to the entropy generation and a reference temperature T_0 by the relationship

$$W_{\text{lost},0} = T_0 \dot{g}.$$

When the entropy generation rate is minimized, so is the lost available work, and maximum work is available per unit mass of the flow rate. From a practical and operational standpoint the implication is that work done on the flow is minimized and that extracted from it maximized.

Despite the attention afforded to entropy generation in convective flows [2, 4, 5] reacting flows have not yet been extensively scrutinized [8, 9]. Arpaci and Selamet [9] examine the entropy generation in premixed flames stabilized above a flat flame burner and relate the tangency condition, i.e. the minimum quench distance, to an extreme of entropy generation which they determine to be inversely proportional to the Peclet number.

In this study the entropy generation equation applicable to chemically reacting flows is described and attention paid to multiphase combustion by considering the burning of particles in a fluid stream. This situation is analogous to that of a burning cloud of coal dust or liquid spray in a flow. The combustion of droplets in a fluid stream is considered, and use made of relationships established previously [10–12] that describe droplet burning, in order to examine the various contributions to the irreversibility. These contributions are compared in order to optimize the flow conditions by minimizing the entropy generation. Though the method is applicable to the combustion of both single droplets and sprays, for the purpose of exposition single droplets burning at small Reynolds numbers are considered.

For burning droplets, in order to minimize the entropy generation, a trade-off must be considered between the *competing irreversibilities* due to the mass loss from the droplet (which dictates the net enthalpy flux into the flow) and drag. While an increase in the transfer number increases the mass loss, it decreases the drag due to a blowing effect, and it is possible to determine a state where the decrease in drag optimally offsets the increase in heat transfer. At this point both the entropy generation and lost available work are minimized.

NOMENCLATURE

A	stream tube area
a	droplet radius
B_M	transfer number
C	concentration of the products
C_D	coefficient of drag
F_D	drag force
$\dot{g}$	entropy generation rate
$\dot{g}'''$	entropy generation rate per unit volume
H	extensive enthalpy
h	intensive enthalpy
k	thermal conductivity
L	heat transfer *to* the particle (e.g. latent heat of vaporization)
$\dot{m}$	mass flow rate
N	mole number
N_s	entropy generation number
P	pressure
Q	heat release from the particle
$\mathbf{q}$	heat flux
R	universal gas constant
Re	Reynolds number
S	extensive entropy
s	intensive entropy
$\mathbf{s}$	directional entropy flux
Sh	Sherwood number
T	temperature
u	velocity
$\mathbf{V}$	volume
V_r	relative velocity
$W_{\text{lost},0}$	lost available work
$\dot{w}$	reaction rate
x	spatial coordinate
$Y_{F,s}$	fuel mass fraction at the particle surface.

Greek symbols

α	chemical potential
Δh	heat of reaction
μ	dynamic viscosity
ρ	density
ϕ	viscous dissipation
Ω	defined in equation (15).

Subscripts

f	subscript on the temperature associated with the particle (e.g. flame temperature)
i	ith species
j, k	repeated indices
in	conditions at the stream tube inlet
0	subscript on the reference temperature
opt	pertaining to optimum conditions
out	conditions at the stream tube outlet
p	conditions pertaining to the particle
∞	ambient conditions.

Superscripts

0	reference state
$\bar{\ }$	indicates that the property is calculated on a per mole basis.

2. ENTROPY GENERATING IN CHEMICALLY REACTING FLOWS

In a fluid flow in which heat, mass transfer and chemical reaction occurs, the conservation of energy is expressed as

$$\frac{\mathrm{D}h}{\mathrm{D}t} = (\nabla \cdot \mathbf{q}) + \mu\phi + (-\Delta h)w + \frac{\mathrm{D}p}{\mathrm{D}t}. \qquad (1)$$

Use of the Gibb's relation

$$\frac{\mathrm{D}h}{\mathrm{D}t} = T\frac{\mathrm{D}s}{\mathrm{D}t} + \frac{1}{\rho}\frac{\mathrm{D}p}{\mathrm{D}t} + \sum_i^n \alpha_i \frac{\mathrm{D}N_i}{\mathrm{D}t} \qquad (2)$$

and the equation of change for the entropy of a fluid

$$\rho\frac{\mathrm{D}s}{\mathrm{D}t} = (\nabla \cdot \mathbf{s}) + \dot{g} \qquad (3)$$

enables description of the local rate of entropy generation [2–4, 8, 9, 13–15] in the form

$$\dot{g}''' = \frac{k}{T^2}\left[\frac{\partial T}{\partial x_j}\right]^2 + \frac{1}{T}\left[\mu\phi + (-\Delta h)\dot{w} - (\rho u_k)\sum_i^n \alpha_i \frac{\partial N_i}{\partial x_k}\right]. \qquad (4)$$

In developing equation (4) from equations (1)–(3) it is noted that by definition the local entropy flux is equal to the local energy flux divided by the local temperature [15], i.e.

$$s_j = q_j/T.$$

For ease of expression, the expressions represented by equations (1) and (4) assume a one-step overall reaction for the combustion of fuel; in the event of simplicity not being desired the term $(-\Delta h)\dot{w}$ is easily replaced by an appropriate representation of the heat release.

For a multiphase flow, such as that involving the flow of a gaseous stream over a burning droplet, the control volume approach [2, 5] is used, and applied to an adiabatic stream tube, in order to develop an expression consistent with equation (4). In the control volume, depicted schematically in Fig. 1, the heat and mass transfer and fluid friction associated with the flow of a gas around a particle are considered. The equations for the conservation of mass, energy and entropy under steady-state conditions are expressed as

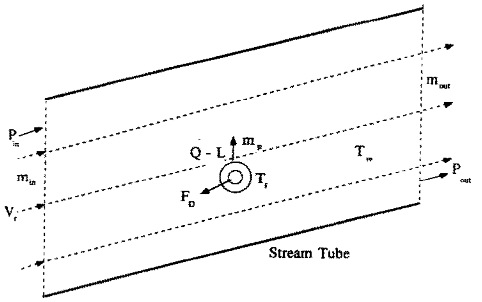

FIG. 1. Schematic diagram of the control volume with external flow over a burning particle.

$$\dot{m}_{\rm in}+\dot{m}_{\rm p} = \dot{m}_{\rm out}$$

$$\dot{m}_{\rm in}h_{\rm in}+Q\dot{m}_{\rm p}-L\dot{m}_{\rm p}+\dot{m}_{\rm p}h_{\rm p}+\mu\phi{\rm V} = \dot{m}_{\rm out}h_{\rm out}$$

$$\dot{m}_{\rm in}s_{\rm in}+\frac{Q\dot{m}_{\rm p}}{T_{\rm f}}-\frac{L\dot{m}_{\rm p}}{T_{\rm f}}+\dot{m}_{\rm p}s_{\rm p} = \dot{m}_{\rm out}s_{\rm out}-\dot{g}. \quad (5)$$

The enthalpy rise within the stream tube has two components: one due to products issuing from the burning particle ($m_{\rm p}h_{\rm p}$), and the other due to the net heat release $m_{\rm p}(Q-L)$ from the particle. Similarly, contributions to the entropy rise in the stream tube arise from the products ($m_{\rm p}s_{\rm p}$), the net heat release, and from the entropy generated by irreversibilities in the flow $\dot{g}$. It is assumed that the heat release occurs at a characteristic temperature $T_{\rm f}$ which, considering the range of temperatures around a burning particle, is an average flow temperature. For the purpose of this study $T_{\rm f}$ is set equal to a flame temperature demonstrative of the combustion phenomena so that, for inclusion in the entropy conservation equation, the net heat release is assigned this temperature. The viscous dissipation appearing in equations (5) is an average term that is assumed to be uniformly distributed over the entire stream tube volume **V**.

The canonical relation

$$\mathrm{d}H = T\,\mathrm{d}S+{\rm V}\,\mathrm{d}P+\sum_{i}^{n}\alpha_i\,\mathrm{d}N_i$$

applied to the flow field under consideration assumes the following form:

$$\dot{m}_{\rm out}h_{\rm out}-\dot{m}_{\rm in}h_{\rm in} = T_\infty[\dot{m}_{\rm out}s_{\rm out}-\dot{m}_{\rm in}s_{\rm in}]$$

$$+\frac{\dot{m}_{\rm in}}{\rho_\infty}[P_{\rm out}-P_{\rm in}]+\alpha_{{\rm p},\infty}[N_{\rm p,out}-N_{\rm p,in}]. \quad (6)$$

The enthalpy and entropy rises in the stream tube are included in the incremental terms in equation (6), one appearing on the left-hand side, and the other being the first contribution to the right-hand side of that expression.

The entropy of the species issuing from the particle is related to its enthalpy and chemical potential by the relation [16]

$$T^{\rm s}_{\rm fp} = \left[h_{\rm p}-\frac{\alpha_{\rm p}}{W_{\rm p}}\right]. \quad (7)$$

The production of species due to mass transfer from the particle can be expressed in terms of the mass loss from the particle and the molecular weight of the species entering the flow as

$$N_{\rm p,out}-N_{\rm p,in} = \frac{\dot{m}_{\rm p}}{W_{\rm p}}. \quad (8)$$

When a mixture composed of several species emanates from the particle, e.g. the products due to a burning droplet or coal particle, equations (6)–(8) are applicable by considering the mean of bulk properties of the product mixture.

If the nonuniformity of the flow is such that the area of the stream tube is almost constant from entry to exit, the force $F_{\rm D}$ required to maintain the control volume stationary is

$$F_{\rm D} = A_{\rm out}(P_{\rm in}-P_{\rm out}) \quad (9)$$

when the fluid velocity is equal to the *relative* velocity of the particle. If the mass loss from the particle is small compared with the gaseous flow into the stream tube, the area of the tube can be expressed as [2]

$$A_{\rm out} = \frac{\dot{m}_{\rm in}}{\rho_\infty V_{\rm r}}. \quad (10)$$

By use of equations (5)–(10) an expression describing the entropy generation in the flow is derived, namely,

$$\dot{g} = \dot{m}_{\rm p}\left[(Q-L+h_{\rm p})\left(\frac{1}{T_\infty}-\frac{1}{T_{\rm f}}\right)\right.$$

$$\left.-\frac{1}{W_{\rm p}}\left(\frac{\alpha_{{\rm p},\infty}}{T_\infty}-\frac{\alpha_{\rm p,f}}{T_{\rm f}}\right)\right]-\frac{F_{\rm D}V_{\rm r}}{T_\infty}+\frac{\mu\phi{\rm V}}{T_\infty}. \quad (11)$$

Equation (11) is the general equation for the entropy generation in an adiabatic stream tube caused by the flow of gas over a particle transferring heat and mass to the flow, and by fluid friction in the tube. Though the term containing the chemical potentials can be further simplified [5] this is unnecessary for the applications that are now considered. It is noted that were the stream tube nonadiabatic a term involving heat transfer at the boundary, and assigned to a boundary temperature, would appear on the right-hand side of equation (11).

3. ENTROPY GENERATION DUE TO DROPLET COMBUSTION

Several correlations, that are of roughly similar form, express the mass loss from a burning droplet [10, 12, 17–19] in terms of the Sherwood number Sh and a transfer number B_M. For sake of illustration a relation from refs. [10, 12] is chosen that describes the mass loss and Sherwood number in the form

$$\dot{m}_{\rm p} = 2\pi\mu_\infty a\,Sh\,B_M$$

$$Sh = \frac{2\ln(1+B_M)}{B_M}+\frac{(1+(1+Re)^{1/3})f_1(Re)-2}{(1+B_M)^{0.7}}. \quad (12)$$

The relationships described by equations (12) assume a Schmidt number of unity. If fuel vapor is absent in the ambient flow the transfer number is related only to the concentration of fuel vapor at the droplet surface [11], i.e.

$$B_M = \frac{Y_{\mathrm{F,s}}}{1 - Y_{\mathrm{F,s}}}. \tag{13}$$

The fuel vapor concentration at the droplet surface is in turn related to the enthalpy of the fuel, net heat release due to its burning, the latent heat of vaporization and conductivity of the liquid fuel, and the ambient temperature and oxidizer concentration [10–12].

Considering the droplet to be a sphere the force on the droplet is described in terms of a drag coefficient where [12]

$$F_{\mathrm{D}} = \tfrac{1}{2}\rho_\infty V_{\mathrm{r}}^2 \pi a^2 C_{\mathrm{D}}$$

$$C_{\mathrm{D}} = \frac{24}{Re} f_2(Re)(1+B_M)^{-0.2}$$

$$f_2(Re) = (1+0.2)Re^{0.63}, \quad Re \geqslant 1$$

$$f_2(Re) = 1, \quad Re < 1. \tag{14}$$

It is instructive to note that equations (12) and (14) indicate a trade-off in terms of the transfer number in the context of equation (11). On the one hand an increase in the transfer number decreases drag, a result which is beneficial in minimizing the entropy generation, but on the other it increases the mass loss from the droplet and thus the irreversibilities associated with the heat and mass transfer. Clearly, the choice of a particular correlation is not of critical significance since the behavior relating the transfer number to the drag and mass loss should prevail in all such expressions, and so it is reasonable to state that, from a thermodynamic standpoint, an investigation to determine an optimum transfer number at which the entropy generation is minimized can be conducted. In fact, perusal of equation (11), in light of equations (12) and (14), elucidates that the only apparent term that can be traded-off in order to minimize the entropy generation is the transfer number. Although the transfer number is a thermodynamic property it can be altered in several ways for a particular fuel. A change in either the ambient temperature, liquid fuel temperature, or ambient oxidizer mass fraction will modify the transfer number; obviously, there are limits in which such changes can be made.

Since the analysis that is developed is uniformly applicable, for the purpose of exposition, and in order to facilitate the study, the low Reynolds number correlations of equations (12) and (14) are assumed to apply to droplets moving with any Reynolds numbers.

An optimum transfer that minimizes the entropy generation in the flow is obtained in a straightforward manner at a given Reynolds number by setting

$$\frac{\partial \dot{g}}{\partial B_M} = 0$$

so that

$$\left[2(1+B_{M,\mathrm{opt}})^{0.2} + ((1+Re)^{1/3} - 1)(1+B_{M,\mathrm{opt}})^{0.5} \times \left[1 - \frac{0.7B_{M,\mathrm{opt}}}{(1+B_{M,\mathrm{opt}})}\right]\right] = \Omega \tag{15}$$

where

$$\Omega = \frac{3}{5}\frac{V_{\mathrm{r}}^2}{L\left(\dfrac{Q+h_{\mathrm{p}}}{L} - 1\right)\left(1 - \dfrac{T_\infty}{T_{\mathrm{f}}}\right) - \dfrac{1}{W_{\mathrm{p}}}\left(\alpha_{\mathrm{p},\infty} - \alpha_{\mathrm{p,f}}\dfrac{T_\infty}{T_{\mathrm{f}}}\right)}.$$

For small Reynolds numbers a description of the optimum transfer number is obtained in the form

$$B_{M,\mathrm{opt}} = \left(\frac{\Omega}{2}\right)^5 - 1. \tag{16}$$

From equation (16) it is observed that the optimum transfer number obtained for small Reynolds numbers is independent of the transport properties of the droplet and its ambient, and of the droplet size. The optimum transfer number increases with increasing relative velocity since the drag is reduced, diminishes with an increase in the net enthalpy rise due to the burning droplet, and increases with a reduction in the ratio of the ambient temperature to the flame temperature. Sufficient care must be used while applying equation (16) so as to maintain the right-hand side of the equation positive.

In a real system there is little control over the transfer number, and the heat release and latent heat of vaporization for a given fuel. The specific enthalpy associated with the products is fixed by the choice of liquid fuel and the stoichiometry related to the oxidizer content in the gas. Equation (16), in this case, describes an optimum relative velocity that minimizes the lost available work for a given gas flow rate and temperature ratio T_∞/T_{f} while the other parameters are maintained constant; the equation assumes the form

$$V_{\mathrm{r}}^2 = \frac{10}{3}(1+B_M)^{0.2}\left[L\left(\frac{Q+h_{\mathrm{p}}}{L} - 1\right)\left(1 - \frac{T_\infty}{T_{\mathrm{r}}}\right) - \frac{\alpha_{\mathrm{p},\infty}}{W_{\mathrm{p}}}\left(1 - \frac{\alpha_{\mathrm{p,f}}}{\alpha_{\mathrm{p},\infty}}\frac{T_\infty}{T_{\mathrm{f}}}\right)\right]. \tag{17}$$

Equations (16) and (17) are applicable to the combustion of single droplets at low Reynolds numbers and to sprays that have associated with them a small mean Reynolds number. In the case of sprays, equation (16) assumes that the droplet loading is dilute so that the entropy rise due to mixing of the products in the flow is negligible and that the ambient temperature remains basically unchanged due to heat release from the spray. The stream tube, in this case, is assumed to contain an ensemble of droplets which are either part or whole of the entire spray, which enables separate analysis of the different regimes of a spray.

To the above analysis the entropy generation due to mixing in the stream tube may be added by use of appropriate empirical relationships. For an ideal gas the chemical potentials are related to the concentrations of a chemical species by the expressions [5, 16, 20]

$$\alpha_p = \alpha_p^0 + RT\ln(P/P^0)$$

$$P = CRT$$

$$\alpha_p^0 = \bar{c}_p(T-T^0)+\bar{c}_p T\ln\frac{T}{T^0} + \bar{h}_p^0 + T\bar{s}_p^0 - T\bar{R}\ln P^0 \quad (18)$$

and a simplified expression for the entropy rise due to mixing (cf. equation (11)) is developed after assuming that the specific heat of the products is constant over a wide range of temperature, so that

$$\frac{1}{W_p}\left(\frac{\alpha_{p,\infty}}{T_\infty}-\frac{\alpha_{p,f}}{T_f}\right)=\left[\frac{R}{W_p}-c_p\right]\ln\frac{T_\infty}{T_f}+\frac{R}{W_p}\ln\frac{C_\infty}{C_f}. \quad (19)$$

If the products and reactants are assumed to have similar properties (such as those of nitrogen, for instance) then the concentration of the products near the particle surface may be assumed to equal that in the flow, i.e. $C_\infty = C_f$. In that case the second term on the right-hand side of equation (19) is zero allowing an estimate for the entropy gain due to mixing to be made on the basis of the mixture thermodynamic properties and the ambient to flame temperature ratio. Assuming the products to have the same specific heat as nitrogen (taken as 0.2911 cal g^{-1} at 1300 K), the same molecular weight (28 g mol^{-1}), and the temperature ratio (T_∞/T_f) to be equal to 300/1800, a numerical value of approximately 0.4 cal g^{-1} K^{-1} is determined for the left-hand side of equation (19). If the fuel has a typical heat release of about 3 kcal g^{-1} clearly the entropy rise due to mixing is small compared with that due to the heat release irreversibility, i.e. $Q(1/T_\infty - 1/T_f)$. Though it is not necessary to drop this term it can be eliminated following the reasoning above, which is done in the subsequent analysis for the sake of brevity and convenient formulation.

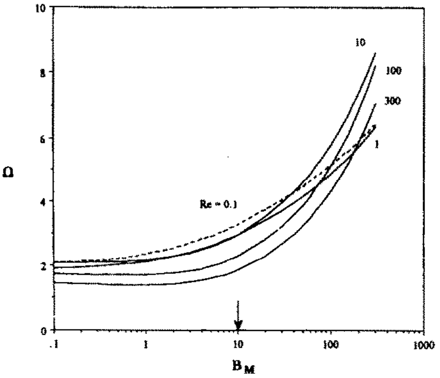

FIG. 2. The variation of the parameter Ω with the transfer number B_M for constant values of the Reynolds number in order to minimize the entropy generation. The dashed line uses the low Reynolds number correlations and the solid ones the high Reynolds number correlations.

The high Reynolds number correlations of equations (12) and (14) can be applied to equation (11) to obtain an implicit expression relating the transfer number at optimum conditions (i.e. when the entropy generation is minimized) and the Reynolds number to the quantity Ω. In Fig. 2 results are presented that show the variation of Ω with $B_{M,\mathrm{opt}}$ for different Reynolds numbers. The arrow marks the condition when $B_{M,\mathrm{opt}}$ equals 10, a value expected for the burning of many hydrocarbon fuels at steady-state conditions. The high Reynolds number correlations were applied for $Re \geqslant 1$ and the low Reynolds number correlations for $Re < 1$. The competing irreversibilities define two regions of optimum operation: one, in which for a given $B_{M,\mathrm{opt}}$, the quantity Ω decreases as the Reynolds number is changed from a value of a tenth to 300 and another where it first decreases and then increases as the Reynolds number is changed over the same range. For a given fuel, when the stoichiometry, ambient temperature and initial fuel temperature are fixed, the only variable in the term Ω is the relative velocity, the results presented in Fig. 2 show that if the optimum transfer number equals 10, the square of the relative velocity at optimum conditions decreases by a factor almost equal to 2 when the Reynolds number is changed from a tenth to 300. This is better explained by considering a dimensionless *entropy generation number* N_s defined as

$$N_s = \frac{\dot{g}}{2\pi\mu_\infty aL\left(\frac{Q+h_p}{L}-1\right)\frac{1}{T_\infty}\left(1-\frac{T_\infty}{T_f}\right)}.$$

Use of equations (11), (12) and (14) yields the following relation for N_s

$$N_s = 2\ln(1+B_M)+[[1+(1+Re)^{1/3}]Re^{0.77}-2] \times\frac{B_M}{(1+B_M)^{0.7}}+\frac{3V_r^2(1+0.2Re^{0.63})(1+B_M)^{-0.2}}{L\left(\frac{Q+h_p}{L}-1\right)\left(1-\frac{T_\infty}{T_f}\right)}. \quad (20)$$

Upon examining equation (20) it is clear that in order to maintain the same value of the entropy generation number, at a fixed transfer number, the quantity Ω must decrease as the Reynolds number is increased. For fixed values of Ω and the Reynolds number an expression similar to that of equation (15) can be obtained and an optimum value of the *entropy generation number* obtained. In Fig. 3 the change in N_s is presented for two cases, i.e. for the quantity Ω possessing values of 3 and 2.5 with the droplet Reynolds number kept at 10. For both cases it is seen that N_s has a minimum value, as defined by equation (20), at an optimum transfer number. The value of $B_{M,\mathrm{opt}}$ changes from approximately 5 to 10 as Ω changes in magnitude from 2.5 to 3 as shown in Fig. 2.

Application of the above analysis to realistic flows reveals some interesting implications since the relative velocity is usually several orders of magnitude less than the heat release when calculated in comparable units (a net heat release of 3 kcal g^{-1} corresponds to a velocity squared of 12.5×10^6 m^2 s^{-2}). This implies that the entropy generation number is always above its minimum value for the usual transfer numbers encountered unless the ratio (T_∞/T_f) is very small. Thus, as determined from Fig. 2, from a thermodynamic perspective these flows should operate at high droplet Reynolds numbers suggesting a large droplet size or a low kinematic viscosity in the gas phase. Some control over Ω may be obtained by varying the temperature ratio between the ambient gas and the flame though significant design restrictions apply.

This point is illustrated by considering the entropy generation number defined by equation (20) and minimizing it with respect to the Reynolds number when Ω and the transfer number are fixed. The optimum Reynolds number to obtain the minimum entropy generation number $N_{s,\min}$ for these conditions is determined by the solution to the following equation:

$$\frac{(1+Re^{1/3})}{Re^{0.553}}\left[0.077\left[1+\frac{1}{(1+Re^{1/3})}\right]+\frac{Re}{3(1+Re)}\right]=0.63\Omega\frac{\sqrt{(1+B_M)}}{B_M}. \quad (21)$$

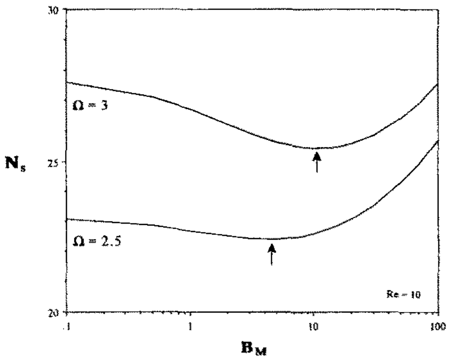

FIG. 3. The entropy generation number N_s plotted as a function of the transfer number B_M showing a minimum corresponding to the position of the optimum transfer number for that flow. The Reynolds number has a value of 10 and the two cases correspond to $\Omega = 2.5$ and 3.

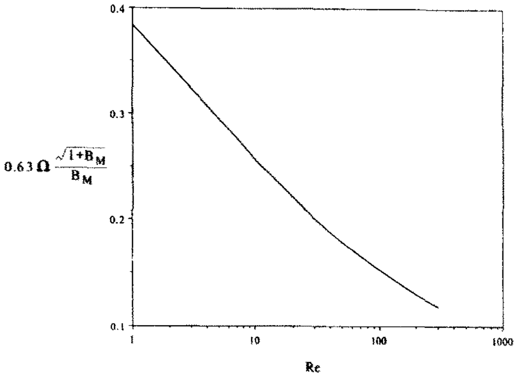

FIG. 4. The change in the optimum Reynolds number that minimizes the *entropy generation number* when the parameter Ω and transfer number B_M are fixed.

In Fig. 4 the variation of the optimum Reynolds number for a fixed right-hand side of equation (20) is presented. The results show that if either Ω or the transfer number becomes smaller in magnitude the optimum Reynolds number which minimizes the *entropy generation number* increases, supporting our earlier inference. Examination of Fig. 4 shows that if, for instance, the Reynolds number has a value equal to 10, N_s is minimized when the right-hand side of equation (20) has a value of about 0.25. In the context of Fig. 2 the implication is that while on the curve corresponding to a constant Reynolds number of 10 there is a unique operating point defining $N_{s,\min}$ where the combination of Ω and B_M prescribed by the right-hand side of equation (20) equals 0.25.

High Reynolds numbers also occur if the relative velocity of the droplets is high which, moreover, enhances gas phase mixing. However, as mentioned above, for reasonable relative velocities such as those expected in practice, the term Ω is small due to the large heat release and the other approach, that of increasing the drop size, should be considered for the reasons stated above. An increase in droplet size may be possible only in certain applications since other considerations, such as those involving residence time and combustor aerodynamics, may suggest selection of a smaller droplet size. In engineering practice the ambient temperature is maintained as high as the material and wall transfer allows [21] which reduces the entropy generation as per the results of equation (11). In practice an increase in the drop transfer number is desirable and accomplished by raising the fuel temperature (within thermal stability limits) and the ambient flow temperature (given the above constrains). Again, for given value of Ω, as the results of Fig. 2 suggest, an increase in the transfer number implies an increase in the Reynolds number.

4. CONCLUSIONS

In this study a second law analysis is applied to reacting flows in order to minimize the entropy generation and lost available work. Specifically, single droplet and spray combustion in a convective environment is considered and the various contributions to the entropy generation are evaluated. It is determined that for droplets burning at low Reynolds numbers the entropy generation is minimized by comparing two terms: one, involving the mass loss from the droplet, and the other the drag force. An expression for an optimum transfer number is obtained which is observed to be directly proportional to the square of the relative velocity and inversely proportional to the heat release and the temperature difference between the droplet and its surrounding flow. The optimum operating condition when the entropy generation and lost available work are minimized is one that provides the maximum net energy output per unit mass of the flow at the combustor exit.

In practical flows the transfer number is fixed and the square of relative velocity is much less than the net heat release, so that in order to optimize these flows from a second law standpoint and operational droplet Reynolds numbers should be large, suggesting a large droplet size and low gas viscosity. Though considerations other than those involving thermodynamics may encourage or inhibit application of the above results in specific circumstances, within constraints enginering practice seems to be consistent with the results of a second law analysis.

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ANALYSE DE LA SECONDE LOI POUR LA COMBUSTION D'UNE GOUTTELETTE AVEC CONVECTION

Résumé—La génération d'entropie due à la combustion de particules dans un écoulement gazeux est considérée. Une analyse est conduite pour minimiser cette génération et, par suite, la perte d'énergie utile. L'optimum des conditions d'écoulement à partir de cette perspective thermodynamiquement avantageuse est déterminé pour une gouttelette en combustion à faible nombre de Reynolds et on obtient un nombre de transfert optimal. Ce nombre ainsi obtenu dépend directement du quarré de la vitesse relative et inversement de l'élévation nette d'enthalpie due à la combustion et au rapport des températures d'ambiance et de flamme. Le quarré de la vitesse relative est une faible fraction de la libération de chaleur et, pour opérer aux conditions thermodynamiques optimales, le nombre de Reynolds de la gouttelette doit être grand, ce qui suggère une grosse taille de goutte et une faible viscosité de gaz. On considère aussi des circonstances se rapportant à la pratique industrielle et on conclut que la pratique est cohérente avec les implications de l'analyse de la seconde loi.

BETRACHTUNG DER KONVEKTIVEN TRÖPFCHENVERBRENNUNG IM HINBLICK AUF DEN ZWEITEN HAUPTSATZ

Zusammenfassung—Es wird die Entropieerzeugung aufgrund der Verbrennung von Partikeln in einer Gasströmung betrachtet, und die einzelnen Beiträge werden miteinander verglichen. Mit Hilfe des zweiten Hauptsatzes wird eine Betrachtung angestellt, um die Entropieerzeugung und damit die verlorene verfügbare Arbeit zu minimieren. Die aufgrund dieser thermodynamisch vorteilhaften Perspektive optimalen Strömungsbedingungen werden für ein brennendes Tröpfchen bei niedriger Reynolds-Zahl und einer daraus resultierenden optimalen Übertragungszahl bestimmt. Die so ermittelte Übertragungszahl hängt direkt vom Quadrat der relativen Strömungsgeschwindigkeit ab und ist umgekehrt proportional zur Netto-Enthalpieerhöhung aufgrund der Verbrennung und zum Verhältnis von Umgebungs- und Flammtemperatur. In realistischen Strömungen, bei denen die Übertragungszahl und die Wärmeabgabe vorgegeben sind, stehen diese Größen in einer bestimmten Beziehung zu der relativen Strömungsgeschwindigkeit und dem Verhältnis von Umgebungs- und Flammtemperatur, um bei optimalen Bedingungen zu arbeiten. Das Quadrat der relativen Strömungsgeschwindigkeit in solchen Strömungen ist ein geringer Bruchteil der Netto-Wärmeabgabe. Für einen Betrieb bei optimalen thermodynamischen Bedingungen muß daher die Reynolds-Zahl der Tröpfchen groß sein, wobei große Tröpfchen und eine geringe Gasviskosität vorgeschlagen werden. Im Hinblick auf die technische Praxis kann gefolgert werden, daß diese innerhalb gewisser Grenzen mit den Ergebnissen aus den Betrachtungen nach dem zweiten Hauptsatz übereinstimmt.

АНАЛИЗ ВТОРОГО ЗАКОНА ТЕРМОЛИНАМИКИ В ЗАДАЧАХ КОНВЕКТИВНОГО ГОРЕНИЯ КАПЕЛЬ

Аннотация—Исследуется производство энтропии за счет горящих частиц в потоке газа. На основании минимизации производства энтропии определяются оптимальные условия течения для горящей капли при низком числе Рейнольдса и оптимальное число переноса, которое прямо пропорционально квадрату относительной скорости и обратно пропорционально суммарному росту энтальпии, обусловленному горением, и отношению температур окружающей среды и пламени. В оптимальных условиях при реальных течениях с фиксированными числом переноса и суммарным тепловыделением эти величины зависят от относительной скорости и отношения температур окружающей среды и пламени. Квадрат относительной скорости таких течений составляет малую часть суммарного тепловыделения, поэтому оптимальные термодинамические условия при большом числе Рейнольдса предполагают крупный размер капли и малую вязкость газа. Предложенная методика используется в конкретных задачах, встречающихся в инженерной практике.